Study of mathematics textbooks at primary and low secondary level

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INTRODUCTION

When studying textbooks, one can adopt different viewpoints: one can explore these objects, to improve them or to select them or one can observe the use of these objects in educational contexts. In the framework of the first approach, we can consider their ideological background, their history, their organization and so on. (see Johnsen (1993) and Iartem's publications for references).

In our research, we adopt a different approach. We study textbooks for themselves without considering their use by teachers or pupils in classroom or at home. We think that textbooks are artefacts which are results of many interwoven constraints. They also reflect ideas at a certain time and constitute a kind of potential curriculum, i.e. describes what can be done in the school. For example, it is reasonable to think that mathematical teachers use textbooks in their classrooms, as least as an exercises databank.

For the moment, textbooks are books and school subjects, activities, exercises have to be described verbally, using sometimes pictures and book materiality have certainly an effect on what can be proposed. Ideas, contexts and so on are evolving and also, with ICT, book materiality is changing, it seems important to study these evolutions, trying to point out the main features that characterize textbooks and their successors.

To do this research, we have chosen to focus on one subject: mathematics. The case of mathematics is certainly a good starting point because mathematical learning requires instruments and learners activities. And mathematics textbooks have deeply changed in one century, from reference texts to catalogue of activities (Baron Bruillard, 1998).

Our research agenda consists in three steps.

First step: to do a descriptive study of French mathematical textbooks in order to acquire a deep understanding of the domain and build analysis grids taking into account the main characteristics;

Second step: use theses grids to analyse textbooks of similar level in other countries;

Third step: build an explicative model emerging from the previous studies.

The study presented here relies on a corpus of French mathematics textbooks at primary and low secondary level, from 1858 to 2003. The aim of the research, more than establishing the grid

mentioned earlier, is to underline links between forms and contents of mathematics textbooks, instruments and technological advances available and national curriculum, pedagogical recommendations. More specific, we investigate the following two questions:

- What is the nature of the link between the evolution of textbooks and the evolution of techniques and instruments available and how do this link operate?
- In the modification of the presentation of a given notion, how do the pedagogical theories and the technological possibilities melt each other?

This text sums up the preliminary work of the descriptive study of mathematics textbooks. We include results concerning the global study of textbooks, their general shape and the mode of presentation of mathematical notions and a more detailed study of the treatment of the area calculus. At the end of the presentation, we will give some assumptions to explain these results.

METHODOLOGY

Previous works (Choppin, 1992; Baron & Bruillard, 1998) give a first overview of interesting features to consider. The idea was to extend these works and to verify their observations in a more exhaustive way. The method is based on the exploration of the corpus. We have access to a previous corpus (in the study concerning area calculus, Bruillard *& al*., 2003). After a first exploration based on forty textbooks, we have completed the corpus with about fifty others textbooks adding new interesting features. To construct the analysis grids we have worked on the area calculus. The area calculus is an interesting notion to conduct a diachronic comparative study for the following reasons:

- the importance devoted to geometry in curriculum varies through historical periods;
- there exist a strongest influence of the technological instruments in geometry ;
- in France, there have been many changes in the presentation of the area calculus during the twentieth century.

The grid is divided in three parts: one for the technical characteristics of the textbook (such as dimensions, typography, colours…), one for the general characteristics of the pedagogical contents (such as lessons or chapters, number of exercises…) and one for the presentation of the area calculus (including items such as previous notion, proof for the formulas…).

For the moment, we have used this grid to point out the main evolutions that we will present in the next three sections.

EVOLUTIONS OF TECHNICAL CHARACTERISTICS

As regards, the evolution of material aspect of textbooks, my observations are analogous to those made by Alain Choppin (1992).

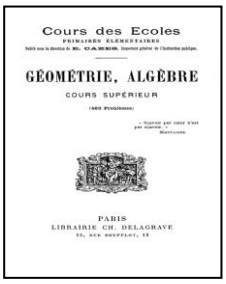
a. Textbooks authors

During the last century, the number of authors participating to the elaboration of a textbook has considerably increased.: from one or two at the beginning of the twentieth century, textbooks are written by redaction staff in the 1980's. Sometimes, only the name of the collection director appears on the cover. The authors are often inspectors, working, nowadays, with a team of mathematic teachers or primary level teachers (for primary textbooks).

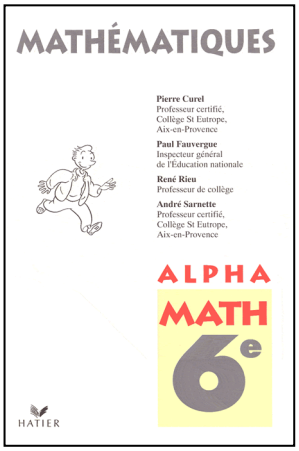
b. Typography

Until the beginning of the twentieth century, textbooks are austere, the text is written with small types and very close with little paragraphs and line jumps. From the end of the nineteen century to the years 1930's, the size of textbook stay little, in general 11 × 18 cm. From 1900, authors and editors begun to take care of the people using textbooks, they adapt their books to the specificities of their young audience. The size of types grows to avoid tiredness link to the reading and the typography varies in the body of the text to help understanding and memorising of results. For examples, the titles are in bold shape, the rules and theorem in slanted shape…

After world war two, the surface of textbooks grows regularly. During the years 50's and 60's, the sizes of textbooks are 15 × 20cm. The sizes grow to square shape, 20 × 20cm, in the 70's. Then, the height increase in the 80's to reach the nowadays size 20 × 29cm, very close to the size of a copy book.



Cover of Cazès's textbook (1900)



Cover of Alphamath textbook (1994)

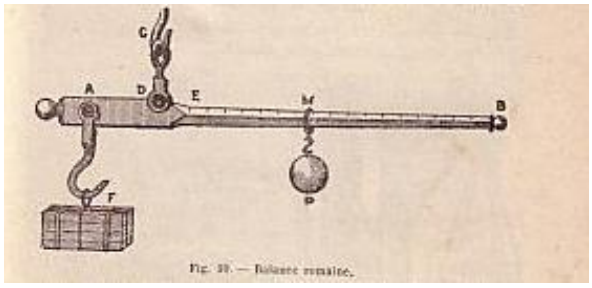
After world war two, the generalisation of new printing techniques and the reduction of production costs allow the use of colours in the body of text. At the beginning, the use of colours is limited to one extra colour, generally green or red to point out a result. Then, during the 70's, several colours are used, in the body of text, or as background or framing colours.

Nowadays, the text is very spaced, with lots of line jumps and frequent changes of colours. Colours are present in the body of the text, of course, but also in margin, heading to indicate the importance of the text.

c. Nature and function of illustrations

Until the 1920's, the only illustrations, excepted geometrical figures, were engravings representing objects that appear in the course, such as, for example, measure instruments. The use of black and white photography increases after 1930. After 1940, textbooks contains illustrations having no link with the mathematical text. However, until the 1970's, the illustrations (drawing and photography) do generally have a link with the mathematic content.

This is after 1985, that the use of funny illustrations is generalised: drawing, humoristic comics, landscapes or places photography sprinkle the mathematic text, having no apparent links with it.



An engraving representing a roman steel-yard (Leyssenne, 1896, p168)

An humoristic illustration (transmath textbook 2000, p 175)

EVOLUTIONS OF PEDAGOGICAL CONTENTS AND STRUCTURE

In this part, I will give some examples of significant evolutions in the general presentation and contents of textbooks.

a. Internal structure of textbooks

In complement of the typographic changes presented in the previous section, there exist a modification of the structure of textbooks at the beginning of the 1980's. Before this decade, textbooks is a book which content is presented in a text written continuously, printed in recto-verso

and divided in lessons or chapters. The numbering used is the "juridique"[1] numbering, that is each notion is numbered increasingly from the beginning to the end of textbook. After the presentation of the knowledge, the authors give some oral exercises based on the mathematical results, applying exercises and problems needing more thought from the pupils. These problems often have a practical or professional background.

After 1980, the textbook is divided in chapters, themselves divided in "rubrics" or "technical cards", organised on a double page. All the chapters of the textbooks have the same structure and the contents of the chapter is partially determined by this organisation which is presented in a directions of use part at the beginning of the textbook. Oral exercises have disappear ,the exercises and problem are classified in function of their difficulty level (basic exercises, extensive exercises, problem "to go further", helping exercises…). To sum up, textbooks are now a catalogue of scholar activities, mathematical results, exercises presented in a succession of rubrics with a lot of colours and illustrations.

b. The introductions

We have studied the introductions or prefaces of textbooks. We can distinguish two periods. Before the 1970's, the introduction is written for teachers. The introduction includes generally a paragraph about the conformity of the textbook with prescribed national curriculum, a long paragraph dealing with the author's pedagogical choices and the justification of some author's specific view. The introduction ends with a skeleton of the textbook with a summary of content. Sometimes, the authors add some pieces of advises for the use of the textbook to the teacher. After 1970, the introduction is written for the pupils : the authors talk directly to the pupil, often using familiar terms. They present to the pupil his new textbook and give him a use guide. One can often find a skeleton of chapters and advises for the use of the different rubrics. This is a little paradox because the textbook does not belong to the pupil.

EVOLUTIONS OF THE AREA CALCULUS PRESENTATION

When looking at the evolution of area calculus presentation in the corpus, we notice some important evolutions corresponding to four major periods.

a. Before 1959 : a presentation with no important changes during 100 years

From 1860 till 1959, the presentation of the area calculus have not really changed. In fact, before 1950, the study of area calculus was dominated by the study of the decimal metric system. There was no geometry in national curriculum before the law on the compulsory primary school in 1882

[1] This is the expression used by Choppin in French (Choppin, 1992)

(D'Enfert, 2003). These curricula mentioned only the study of the metric system. Since 1882, geometry is present in the national curriculum. However, a large part of curriculum is reserved to the metric system. The chapters devoted to geometry, especially the area calculus, are preceded, at least until the 1950's, by a chapter aiming to study the metric system. This attachment to the metric system in France has historical roots, because it is the French Revolution that takes a great care of the construction and the use of the metric system. It is one of the symbols of the French Republic unity.

During this period, the chapter dealing with the area calculus begins with the definition of the main surface's unit of measure, a square of one meter length named a square meter. Then, the proof of the centesimal law, which gives the relation between two consecutive units of measures, is given by the way of a square divided in 100 small squares. Next, an important part of the chapter, consists in a specific study of land measures (*centiare*, *are* and *hectare*). Then, the authors insist on the fact that there is no real or effective measure of a surface: the geometry gives tools using length measures for computing areas. The formulas giving the areas of basic geometric figures are presented, often with an illustration showing the corresponding figure. These formulas are obtained from a decomposition of the figure in simplest figures for which the area's formulas are know.

b. In 1959, a teaching reform

In 1959, an important teaching reform takes place in France. The age of compulsory school is prolonged to the age of sixteen, and most important, the primary school becomes the first degree, divided in five years. After the first degree, the pupils go to the general or professional second degree.

After this reform, the area calculus is mainly presented in grade 6 and 7, corresponding in France to the first two years of the low secondary school. The course still keep is traditional aspect. However, some innovations can be found in the exercises: millimetre paper is used for counting the squares included in the figure to obtained the area, the area is also computed by used weighing the surface and comparison with a gauge.

c. The period of "modern mathematics"

In 1968, with the apparition of modern mathematics in the curriculum of 1968 and 1977, the aspect of the course on areas calculus have radically changed. The study of the metric system is not systematically done, and the study of squaring comes before the area notion. The notion of surface is presented with the vocabulary of sets theory and the notion of surface's area is introduced by the mean of equivalence classes. The geometric decomposition of figures disappears in favour of counting.

d. The present period

Nowadays, from 1980 till 2003, the presentation of the area notion is still based on squaring and counting, nevertheless, the sets theory is no more used. However, one can find some geometrical decompositions in the course for some basic figures, and some deepening exercises includes geometrical decompositions.

CONCLUSION

To try to explain these evolutions, we can point out the followings ideas.

The impact of the instruments used can not be neglected. Thus, as regards the area calculus, the apparition of a new technological instrument in classrooms, the millimetre paper, has induced modifications in the treatment of the area notion. This instrument, is indeed often used by pupils to evaluate the area of rather small figures (hence a diminution of the size of figures in textbooks) in detriment to activities of geometrical nature. Likewise, the generalisation of the use of calculator can explain the diminution of calculating exercises. The computers can give raise to new activities too…

The evolution of the society can explain too some of the evolutions that I have pointed out. for example, the diminution of the number of farmers in the 1970's can explain the fact that the study of agrarian measures units and the calculus of a fields area have almost disappeared.

The democratisation of teaching, the lengthening of the duration of compulsory school and the decrease of the secondary sector in benefit of the tertiary sector can explain the diminution of number of practical or professional exercises and problems.

Last, the national curriculum insists on the fact that pupils must have a much more reflexive work, which incite the author of textbooks to write preparatory activities and to reduce the number of exercises involving only the use of a formula.

So, these first results illustrate our research point of view, testifying evolutions with multi-dimensional explanations. The analysis grid, not presented in that paper, is now operative.

Our current perspective is to study textbooks from other countries. The expected corpus will include, for each of the studied country, textbooks from periods corresponding to changes in the education system of the country, official texts on educations, curriculum, texts about pedagogical trends.

In this framework, we want to try to understand the incidence of several factors such as the culture of the country, the teaching methods, the organisation of editors, the choice of authors on contents and the form of textbooks. In particular, we want to investigate the following questions:

- What are the significant differences between French textbooks and other countries textbooks ?
- Is there some differences in the evolution of the general aspect of textbooks?
- For a given mathematical notion, are the invariants in the treatment of the notion the same?

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